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1□2021·□□·□□□□□□□□□□□□□□□□ $f(x)=ne^x, g(x)=\ln x+1.$

$$\frac{f(x)}{g(x)} = m$$
$$\exists x \in \mathbb{R} \text{ such that } f(x) > g(x) + 1 \text{ for all } m \in \mathbb{N}.$$
$$\Pr[\text{all } 1] \leq \frac{1}{e} \Pr[\text{all } 2] \leq \frac{1}{e} \Pr[\text{all } 1].$$

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$$\boxed{1} \quad f(x) = g(x) \quad \boxed{m = \frac{\ln x + 1}{e^x}} \quad \boxed{f(x) = g(x)} \quad \boxed{m = \frac{\ln x + 1}{e^x}} \quad \boxed{h(x) = \frac{\ln x + 1}{e^x}} \quad \boxed{h(x)}$$

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$$\square \square \square \square f(x) > g(x) + 1 \square \square \square \square n e^x - \ln x - 1 > 0 \square \square \square \square x = 1 \square \square n e > \ln 1 + 2 \square \square \square \square m > \frac{2}{e} \square \square m \geq 1. \square \square m = 1 \square$$

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$$f(x) = g(x) \cdot e^{m \ln x + 1} = \frac{\ln x + 1}{e^x}$$
$$f(x) - g(x) = m = \frac{\ln x + 1}{e^x}.$$
$$h(x) = \frac{\ln x + 1}{e^x} \quad h(x) = \frac{\frac{1}{x} - \ln x - 1}{e^x}.$$
$$k(x) = \frac{1}{x} - \ln x - 1 = \left(\frac{1}{x} - 1 \right) - \ln x$$
$$x > 1 \implies \frac{1}{x} - 1 < 0, \ln x > 0 \implies k'(x) < 0 \implies 0 < x < 1 \implies \frac{1}{x} - 1 > 0, \ln x < 0 \implies k'(x) > 0$$

$h(x)$ (0,1) (1,+∞)

$$\lim_{x \rightarrow 0} h(x) = h(1) = \frac{1}{e} \quad \lim_{x \rightarrow 0} f(x) = -\infty$$

$$m \leq \frac{1}{e}.$$

$$f(x) > g(x) + 1 \iff me^x - \ln x - 1 > 0.$$

$$x=1 \implies me > \ln 1 + 2 \iff m > \frac{2}{e}$$

$$m \text{ 为任意实数 } m=1 \text{ 时成立}.$$

$$m=1 \implies m(x)=e^x-\ln x-2, m'(x)=e^x-\frac{1}{x} \implies m'(x) \in (0,+\infty) \text{ 恒成立}.$$

$$m(1) > 0, m\left(\frac{1}{2}\right) < 0 \implies \text{存在 } x_0 \in \left(\frac{1}{2}, 1\right) \text{ 使得 } m(x_0) = 0$$

$$e^{x_0} - \frac{1}{x_0} = 0$$

$$m(x) \in (0, x_0) \text{ 恒成立 } (x_0, +\infty) \text{ 恒成立} \implies m(x)_{\min} = m(x_0) = e^{x_0} - \ln x_0 - 2 = \frac{1}{x_0} + x_0 - 2 > 0$$

$$m=1 \implies f(x) > g(x) + 1.$$

$$m \text{ 为任意实数 } 1.$$

$$m \text{ 为任意实数}$$

$$x=1 \implies me > \ln 1 + 2 \implies m > \frac{2}{e}$$

$$m=1 \text{ 时成立}.$$

$$2021 \cdot \sin x \cdot \ln x \cdot (x^2 - ax - 1)$$

$$x \in [0, 1] \implies f(x) \geq g(x+1)$$

$$x \in [0, 1] \implies e^{f(x)} + h(x) - g(x) > 0 \implies a \text{ 为任意实数}$$

$$1 \text{ 为任意实数 } 2 \cdot 2$$

$$m \text{ 为任意实数}$$

$$1 \text{ 为任意实数 } x_0 \in (0, 1) \implies F(x_0) = 0 \implies F'(x) > 0 \implies (0, 1) \text{ 恒成立 } F(x) \text{ 为任意实数}$$

$$F(x) \geq 0$$

$$a \leq 2 \quad e^{\sin x} + x^2 - ax - 1 - \ln x \geq e^{\sin x} + x^2 - 2x - 1 - \ln x$$

$$H(x) = e^{\sin x} + x^2 - 2x - 1 - \ln x > 0 \quad (0,1]$$

$$F(x)$$

$$F(x)$$

$$F(x) = \sin x \cdot \ln(x+1) \quad (0 \leq x \leq 1) \quad F'(x) = \cos x \cdot \frac{1}{x+1}$$

$$F'(x) = \frac{1}{(x+1)^2} - \sin x \quad x \in [0,1]$$

$$F'(x) \in [0,1] \quad \frac{1}{4} - \sin 1 < 0 \quad F'(x) < F'(0) = 1$$

$$x_0 \in (0,1) \quad F'(x_0) = 0$$

$$F'(x) \in (0, x_0) \quad (x_0, 1)$$

$$F(1) = -\frac{1}{2} + \cos 1 > -\frac{1}{2} + \cos \frac{\pi}{3} = 0 \quad F(0) = 0$$

$$F(x) > 0 \quad (0,1) \quad F(x) \in [0,1]$$

$$F(x) \geq F(0) = 0 \quad F(x) \geq 0$$

$$f(x) \geq g(x+1)$$

$$x \in (0,1] \quad e^{f(x)} + h(x) \cdot g(x) > 0$$

$$e^{\sin x} + x^2 - ax - 1 - \ln x > 0$$

$$x=1 \quad e^{\sin 1} > a$$

$$\sin 1 > \ln 2 \quad 2 = e^{\ln 2} < e^{\sin 1} < e < 3$$

$$\square\square a e^{\sin x} + x^2 - ax - 1 - \ln x > 0 \square\square\square\square a \leq 2$$

$$\square\square e^{\sin x} + x^2 - ax - 1 - \ln x \geq e^{\sin x} + x^2 - 2x - 1 - \ln x$$

$$\square\square\square\square H(x) = e^{\sin x} + x^2 - 2x - 1 - \ln x > 0 \square\square\square\square (0,1] \square\square\square\square\square\square.$$

$$\square\square 1 \square\square \sin x > \ln(x+1) \square\square e^{\sin x} > x+1$$

$$\square H(x) > x+1 + x^2 - 2x - 1 - \ln x = x^2 - x - \ln x$$

$$\square G(x) = x^2 - x - \ln x \square_{x \in (0,1]} \square\square G(x) = 2x - 1 - \frac{1}{x} = \frac{(2x+1)(x-1)}{x} \leq 0 \square$$

$$\square\square G(x) \square (0,1] \square\square\square\square\square\square\square\square G(x) \geq G(1) = 0 \square\square\square H(x) > 0 \square_{x \in (0,1]} \square\square\square\square.$$

$$\square\square\square\square\square a \square\square\square\square\square 2 \square$$

$$3. \square\square\square\square\square\square\square a \square\square\square e^x - ax \geq x^2 \ln x \square\square\square x > 0 \square\square\square\square\square\square a \square\square\square\square.$$

$$\square:\square\square e^x - ax \geq x^2 \ln x \square\square\square x > 0 \square\square\square\square\square x=1 \square\square a \leq e \square\square a \square\square\square 1 \square 2$$

$$\square\square a=2 \square\square\square\square\square\square\square\square\square g(x) = \frac{e^x}{x^2} - \frac{2}{x} - \ln x, g'(x) = \frac{(x-2)(e^x - x)}{x^3}$$

$$g(x) \square (0,2) \square\square\square\square\square (2,+\infty) \square\square\square\square\square g(x) \geq g(2) = \frac{1}{4}(e^2 - 4 - 4\ln 2) > 0$$

$$\square a=2 \square\square\square\square\square\square\square\square\square\square a \square\square\square 2.$$

$$4. \square_{x>2, k<\frac{x\ln x+x}{x-2}}, \square\square k \square\square\square\square\square\square.$$

$$\square:\square f(x) = \frac{x\ln x+x}{x-2} \square\square\square k < f(e^2) = \frac{3e^2}{e^2-2} \in (4,5)$$

$$\square\square k \square\square\square\square\square\square\square\square\square 4 \square\square\square k=4 \square\square\square\square$$

$$\square \ln x \geq 1 - \frac{1}{x} \Rightarrow \ln \frac{x}{e^2} \geq 1 - \frac{e^2}{x} \ln x \geq 3 - \frac{e^2}{x} \square$$

$$\square \square \quad f(x) = \frac{x \ln x + x}{x-2} \geq \frac{x \left(3 - \frac{e^2}{x} \right) + x}{x-2} = 4 + \frac{8 - e^2}{x-2} > 4$$

$$5. \square \square \square \quad x e^x - 2x + k > 0 \quad \square [0, +\infty) \quad \square \square \square \square \square \square \square \square \square \quad k$$

$$\square : \square \quad g(x) = x e^x - 2x + k \quad \square \square \square \square \quad g(0) > 0 \quad \square \square \square \square \square \square \square \quad k > 0 \quad \square \quad k = 1 \quad \square \square \square \square$$

$$\square \square \quad k = 1 \quad \square \square \quad x e^x - 2x + 1 > 0 \quad \square \square \square$$

$$\square \quad e^x \geq 1 + x \Rightarrow x e^x - 2x + 1 \geq x(x+1) - 2x + 1 > 0$$

$$6 \square \square 2019 \bullet \square \square \square \square \square \square \square \square \quad f(x) = (x-1)e^x - \frac{a}{2}x^2 \quad \square \square \square \quad a \in \mathbb{R} \quad \square$$

$$\square \square \square \square \quad f(x) \quad \square \square \square \square \square \square \quad x \quad \square \square \square \square \square \square \square \square \square \square \quad a \quad \square \square \square \square \square \square \square \square \square \square$$

$$\square \square \square \square \square \square \square \square \quad a \quad \square \square \square \square \square \square \quad x_1 \in \mathbb{R} \quad x_2 \in (0, +\infty) \quad \square \square \square \square \quad f(x_1 + x_2) - f(x_1 - x_2) > -2x_2 \quad \square \square \square \square$$

$$\square \square \square \square \square \square \square \square \quad f'(x) = x e^x - ax \quad \square$$

$$\square \square \square \square \quad f(x) \quad \square \square \square \square \quad x \quad \square \square \square \square \square \quad (t, 0) \quad \square$$

$$\square \square \quad \begin{cases} f(t) = 0 \\ f'(t) = 0 \end{cases} \quad \square \square \quad \begin{cases} (t-1)e^t - \frac{a}{2}t^2 = 0 \\ te^t - at = 0 \end{cases} \quad \square$$

$$\square \square \quad t \neq 0 \quad \square \quad e^t = a > 0 \quad \square \square \square \square \square \quad (t-1)e^t - \frac{a}{2}t^2 = 0 \quad \square \square \square \quad t^2 - 2t + 2 = 0 \quad \square$$

$$\square \quad \Delta = -4 < 0 \quad \square \therefore \square \square \quad t^2 - 2t + 2 = 0 \quad \square \square \square$$

$$\square \square \square \quad a \quad \square \square \square \square \square \square \quad f(x) \quad \square \square \square \square \square \square \square \quad x \quad \square \square \square \square$$

$$\square \square \square \square \square \square \square \quad f(x_1 + x_2) - f(x_1 - x_2) > (x_1 - x_2) - (x_1 + x_2)$$

$$\Leftrightarrow f(x_1 + x_2) + (x_1 + x_2) > f(x_1 - x_2) + (x_1 - x_2) \quad \square \square \square \square$$

$$\square \quad g(x) = f(x) + x \quad \square \square \square \square \square \square \square \quad g(x_1 + x_2) > g(x_1 - x_2) \quad \square$$

$$\square \square \mathcal{G}(x_1 + x_2) > \mathcal{G}(x_1 - x_2) \square \square \square x_1 \in R \square x_2 \in (0, +\infty) \square \square \square \square \square \square \mathcal{G}(x) = (x-1)e^x - \frac{a}{2}x^2 + x \square R \square \square \square \square \square \square$$

$$\therefore \mathcal{G}'(x) = xe^x - ax + 1 \dots 0 \square R \square \square \square \square \square \square$$

$$\square \mathcal{G}' \square 1 \square = e - a + 1 \dots 0 \square \square a, e + 1 \square$$

$$\therefore \mathcal{G}'(x) \dots 0 \square R \square \square \square \square \square \square \square \square \square a, e + 1 \square$$

$$\square \square \square \square \square \square a = 3 \square \square xe^x - 3x + 1 \dots 0 \square \square \square \square$$

$$\square h(x) = e^x - x - 1 \square \square h(x) = e^x - 1 \square$$

$$\square x < 0 \square \square h(x) < 0 \square \square x > 0 \square \square h(x) > 0 \square$$

$$\therefore h(x)_{\min} = 0 \square \square \forall x \in R \square e^x \geq x + 1 \square$$

$$\square \square \square \square x \dots 0 \square \square xe^x \dots x^2 + x \square \square xe^x - 3x + 1 \dots x^2 - 2x + 1 = (x-1)^2 \dots 0 \square$$

$$\square x < 0 \square \square e^x < 1 \square \square xe^x - 3x + 1 = x(e^x - 3 + \frac{1}{x}) > 0 \square \therefore xe^x - 3x + 1 \dots 0 \square \square \square \square$$

$$\square \square \square a \square \square \square \square \square \square \square 3 \square$$

$$7. \square 2021 \bullet \square \square \square \square \square \square \square \square f(x) = e^x + a \cos x - \sqrt{2}x - 2 \square f'(x) \square f(x) \square \square \square \square \square$$

$$\square 1 \square \square \square f'(x) \square \square \square (0, \frac{\pi}{2}) \square \square \square \square \square \square \square \square$$

$$\square 2 \square \square x \in [-\frac{\pi}{2}, 0] \square \square f(x) \dots 0 \square \square \square \square \square \square \square a \square \square \square \square \square$$

$$\square \square \square \square \square \square \square 1 \square \square f(x) = e^x + a \cos x - \sqrt{2}x - 2 \square \square f'(x) = e^x - a \sin x - \sqrt{2} \square$$

$$\square \mathcal{G}(x) = e^x - a \sin x - \sqrt{2} \square \square \mathcal{G}'(x) = e^x - a \cos x \square$$

$$\because x \in (0, \frac{\pi}{2}) \square \therefore e^x > 1 \square 0 < \cos x < 1 \square$$

$$\square a, 1 \square \square \mathcal{G}'(x) > 0 \square \mathcal{G}(x) \square \square \square \square \square \square f(x) \square \square \square (0, \frac{\pi}{2}) \square \square \square \square \square \square$$

$$\square a > 1 \square \square g'(x) = e^x + a \sin x \square \square x \in (0, \frac{\pi}{2}) \square \square g'(x) > 0 \square$$

$$\square g(x) \square \square (0, \frac{\pi}{2}) \square \square \square \square \square \square g(0) = 1 \square a < 0 \square g(\frac{\pi}{2}) = e^{\frac{\pi}{2}} > 0 \square$$

$$\square \square x_0 \in (0, \frac{\pi}{2}) \square \square g(x_0) = 0 \square \square x \in (0, x_0) \square \square g'(x) < 0 \square g(x) \square \square$$

$$x \in (x_0, \frac{\pi}{2}) \square \square g(x) > 0 \square g(x) \square \square \square \square \square \square x = x_0 \square g(x) \square \square \square \square \square \square$$

$$\square \square f(x) \square \square \square \square (0, \frac{\pi}{2}) \square \square \square 1 \square \square \square \square \square \square \square \square \square \square \square \square$$

$$\square \square \square a, 1 \square \square f(x) \square \square \square \square (0, \frac{\pi}{2}) \square \square \square \square \square \square$$

$$\square a > 1 \square \square f(x) \square \square \square \square (0, \frac{\pi}{2}) \square \square \square 1 \square \square \square \square \square \square \square \square \square \square \square \square$$

$$\square 2 \square \square x \in [-\frac{\pi}{2}, 0] \square \square f(x) \dots 0 \square \square \square \square f(0) = 1 + a - 2 \dots 0 \square \square a \dots 1 \square$$

$$\square \square \square a \dots 1 \square \square f(x) \dots 0 \square \square x \in [-\frac{\pi}{2}, 0] \square \square \square \square$$

$$\therefore x \in [-\frac{\pi}{2}, 0] \square \square 0, \cos x, 1 \square \square a \dots 1 \square \square f(x) = e^x + a \cos x - \sqrt{2}x - 2 \dots e^x + \cos x - \sqrt{2}x - 2 \square$$

$$\square h(x) = e^x + \cos x - \sqrt{2}x - 2 \square \square x \in [-\frac{\pi}{2}, 0] \square \square h(x) = e^x - \sin x - \sqrt{2} \square$$

$$\square \varphi(x) = e^x - \sin x - \sqrt{2} \square \square \varphi'(x) = e^x - \cos x \square \varphi''(x) = e^x + \sin x \square \square [-\frac{\pi}{2}, 0] \square \square \square \square \square \square$$

$$\square \varphi'(-\frac{\pi}{3}) = e^{-\frac{\pi}{3}} - \frac{\sqrt{3}}{2} < e^1 - \frac{\sqrt{3}}{2} < 0 \square \square \varphi'(x) \square \square [-\frac{\pi}{2}, -\frac{\pi}{3}] \square \square \square \square \square \square$$

$$\square \varphi'(-\frac{\pi}{2}) = e^{-\frac{\pi}{2}} > 0 \square \varphi'(-\frac{\pi}{3}) = e^{-\frac{\pi}{3}} - \frac{1}{2} < e^1 - \frac{1}{2} < 0 \square$$

$$\square \square x_1 \in (-\frac{\pi}{2}, -\frac{\pi}{3}) \square \square \varphi'(x_1) = 0 \square \square x \in (-\frac{\pi}{2}, x_1) \square \square \varphi'(x) > 0 \square h(x) \square \square$$

$$x \in (x_1, 0) \square \square \varphi'(x) < 0 \square h(x) \square \square \square \square \square \square x = x_1 \square \square h(x) \square \square \square \square \square \square h(x)_{\max} = h(x_1) \square$$

$$\therefore \varphi'(x) = 0 \Rightarrow e^x = \cos x \therefore h(x)_{\max} = h(x) = \cos x - \sin x - \sqrt{2} = \sqrt{2} \cos(x + \frac{\pi}{4}) - \sqrt{2}, 0$$

$$h(x) \text{ on } x \in [-\frac{\pi}{2}, 0] \Rightarrow h(x) \dots h(0) = 0 \Rightarrow f(x) \dots 0$$

$$\text{on } x \in [-\frac{\pi}{2}, 0] \Rightarrow f(x) \dots 0 \text{ on } a \text{ on } 1$$

$$8 \text{ on } f(x) = ae^x - x^2 (a \in \mathbb{R}) \text{ on } e \approx 2.71828$$

$$1 \text{ on } a = 1 \text{ on } f(x) \text{ on } \frac{1}{2}$$

$$2 \text{ on } f(x) > \frac{1}{2} \ln(x+1) + \cos x \text{ on } x \in [0, +\infty) \text{ on } a$$

$$1 \text{ on } a = 1 \text{ on } f(x) = e^x - x^2 \text{ on } f(x) = e^x - 2x$$

$$g(x) = f(x) = e^x - 2x \text{ on } g'(x) = e^x - 2 \text{ on } g'(x) = 0 \text{ on } x = \ln 2$$

$$g(x) \text{ on } (-\infty, \ln 2) \text{ on } (\ln 2, +\infty)$$

$$g(x) \text{ on } g(\ln 2) = 2 - 2\ln 2 > \frac{1}{2} \text{ on } f(x) > \frac{1}{2} \text{ on } x \in \mathbb{R}$$

$$f(x) \text{ on } \frac{1}{2}$$

$$2 \text{ on } x \in [0, +\infty) \text{ on } \ln(x+1), x \text{ on } e^x \dots x^2 + \frac{1}{2}x + 1$$

$$h(x) = \ln(x+1) - x \text{ on } h(x) = \frac{1}{x+1} - 1 = -\frac{x}{x+1} \text{ on } h(x) = 0 \text{ on } x = 0$$

$$h(x) \text{ on } (-1, 0) \text{ on } (0, +\infty)$$

$$h(x), h(0) = 0 \text{ on } \ln(x+1), x$$

$$p(x) = e^x - x^2 - \frac{1}{2}x - 1 \text{ on } p(x) = e^x - 2x - \frac{1}{2} = m(x) \text{ on } m(x) = e^x - 2$$

$$m(x) = 0 \text{ on } x = \ln 2$$

$$\square m(x) \square\square\square (-\infty, \ln 2) \square\square\square\square\square\square (\ln 2, +\infty) \square\square\square$$

$$\square m(x) \dots m(\ln 2) > 0 \square\square p(x) > 0 \square p(x) \square\square\square$$

$$\square p(x) \dots p(0) = 0 \square\square e^x \dots x^2 + \frac{1}{2}x + 1 \square\square x \in [0, +\infty) \square\square$$

$$\square f(x) > \frac{1}{2} \ln(x+1) + \cos x \square\square\square\square\square x \in [0, +\infty) \square\square\square\square$$

$$\square f(0) > \frac{1}{2} \ln(0+1) + \cos 0 = 1 \square\square a > 1 \square$$

$$\square a > 1 \square\square f(x) - \frac{1}{2} \ln(x+1) - \cos x$$

$$= ae^x - x^2 - \frac{1}{2} \ln(x+1) - \cos x > e^x - x^2 - \frac{1}{2} \ln(x+1) - \cos x$$

$$\dots x^2 + \frac{1}{2}x + 1 - x^2 - \frac{1}{2} \ln(x+1) - \cos x = \frac{1}{2}x - \frac{1}{2} \ln(x+1) + 1 - \cos x$$

$$\dots \frac{1}{2}x - \frac{1}{2}x + 1 - \cos x = 1 - \cos x \cdot 0 \square$$

$$\square f(x) > \frac{1}{2} \ln(x+1) + \cos x \square\square\square\square\square\square x \in [0, +\infty) \square\square\square\square$$

$$\square\square\square a \square\square\square\square\square\square (1, +\infty) \square$$

$$9\square\square\square f(x) = \sin x - ax + 1 \square$$

$$\square 1\square a = \frac{1}{2} \square\square f(x) \square\square\square\square\square\square\square$$

$$\square 2\square\square f(x) \dots \cos x \square\square x \in [0, \pi] \square\square\square\square\square\square\square\square\square a \square\square\square\square\square\square\square$$

$$\square 3\square\square\square\square g(x) = f(x) + ax - 1 \square\square\square\square g\left(\frac{\pi}{15}\right) + g\left(\frac{2\pi}{15}\right) + g\left(\frac{3\pi}{15}\right) + \dots + g\left(\frac{8\pi}{15}\right) \dots \frac{2\sqrt{2}}{5} \square$$

$$\square\square\square\square\square\square\square\square 1\square a = \frac{1}{2} \square\square f(x) = \sin x - \frac{1}{2}x + 1 \square\square f(x) = \cos x - \frac{1}{2} \square$$

$$\square -\frac{\pi}{3} + 2k\pi < x < \frac{\pi}{3} + 2k\pi \square\square k \in \mathbb{Z} \square\square f(x) > 0 \square$$

$$\frac{\pi}{3} + 2k\tau < x < \frac{5\pi}{3} + 2k\tau \quad k \in \mathbb{Z} \quad f(x) < 0$$

$$f(x) \text{ sur } \left(-\frac{\pi}{3} + 2k\tau, \frac{\pi}{3} + 2k\tau\right) \quad k \in \mathbb{Z}$$

$$f(x) \text{ sur } \left(\frac{\pi}{3} + 2k\tau, \frac{5\pi}{3} + 2k\tau\right) \quad k \in \mathbb{Z}$$

$$2 \text{ sur } [a x + \cos x - \sin x - 1, 0]$$

$$h(x) = ax + \cos x - \sin x - 1 \quad \begin{cases} h(0), 0 \\ h(\pi), 0 \\ h(\frac{\pi}{2}), 0 \end{cases} \quad a, \frac{2}{\pi}$$

$$y = ax + \cos x - \sin x - 1 \quad a$$

$$\varphi(x) = \frac{2}{\pi} x + \cos x - \sin x - 1$$

$$\forall a, \frac{2}{\pi} \quad \forall x \in [0, \pi] \quad h(x), \varphi(x)$$

$$\varphi(x) = \frac{2}{\pi} x + \cos x - \sin x - 1, 0$$

$$\varphi'(x) = \frac{2}{\pi} - \sin x - \cos x = \frac{2}{\pi} - \sqrt{2} \sin(x + \frac{\pi}{4})$$

$$1^\circ \quad x \in (0, \frac{\pi}{2}) \quad \sin x + \cos x = \sqrt{2} \sin(x + \frac{\pi}{4}) \in (1, \sqrt{2}]$$

$$\varphi'(x) = \frac{2}{\pi} - \sin x - \cos x < \frac{2}{\pi} - 1 < 0 \quad \varphi(x) \text{ sur } (0, \frac{\pi}{2}) \quad \varphi(x) = 0$$

$$\varphi(x) < 0$$

$$2^\circ \quad x \in (\frac{3\pi}{4}, \pi) \quad \varphi'(x) = \frac{2}{\pi} - \sin x - \cos x = \frac{2}{\pi} - \sqrt{2} \sin(x + \frac{\pi}{4}) > 0$$

$$3^\circ \quad x \in (\frac{\pi}{2}, \frac{3\pi}{4}) \quad \varphi'(x) = \frac{2}{\pi} - \sin x - \cos x = \frac{2}{\pi} - \sqrt{2} \sin(x + \frac{\pi}{4})$$

$$\varphi'(\frac{\pi}{2}) < 0 \quad \varphi'(\frac{3\pi}{4}) > 0 \quad (\frac{\pi}{2}, \frac{3\pi}{4}) \quad x \quad \varphi'(x_0) = 0$$

$$\square \quad x \in (0, x_0) \quad \square \quad \varphi'(x) < 0 \quad \square \quad x \in (x_0, \pi) \quad \square \quad \varphi'(x) > 0 \quad \square$$

$$\square \quad \varphi(x) \quad \square \quad x \in (0, x_0) \quad \square \square \square \square \square \square \quad x \in (x_0, \pi) \quad \square \square \square \square \square \square$$

$$\therefore \varphi(0) = 0 \quad \square \quad \varphi(\pi) = 0 \quad \square \quad \varphi(x_0) < 0 \quad \square$$

$$\therefore \varphi(x) < 0 \quad \square \square \square \square \quad f(x), \quad \varphi(x) < 0 \quad \square \square \square \square$$

$$\therefore a, \frac{2}{\pi} \quad \square$$

$$\square \square \square \square \square \square \square \square \square \quad \sin x - \cos x \cdot \frac{2}{\pi} x - 1 \Rightarrow \sqrt{2} \sin(x - \frac{\pi}{4}) \cdot \frac{2}{\pi} x - 1 \Rightarrow \sin(x - \frac{\pi}{4}) \cdot \frac{\sqrt{2}}{\pi} x - \frac{\sqrt{2}}{2} \quad \square$$

$$g(x) = \sin x \quad \square \quad x - \frac{\pi}{4} = \frac{k\pi}{15} \quad \square \quad x = \frac{4k+15}{60}\pi \quad \square \quad k=1 \quad \square \square \square \quad \cdots \quad \square \square \square$$

$$\square \square \quad \sin \frac{k\pi}{15} \cdots \frac{\sqrt{2}}{\pi} \times \frac{(4k+15)\pi}{60} - \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{60} (4k-15) \quad \square$$

$$\square \quad \sum_{k=1}^8 \sin \frac{k\pi}{15} \cdots \frac{\sqrt{2}}{60} \sum_{k=1}^8 (4k-15) = \frac{\sqrt{2}}{60} (4 \times \frac{8 \times (1+8)}{2} - 15 \times 8) = \frac{2\sqrt{2}}{5} \quad \square$$

$$\square \quad g(\frac{\pi}{15}) + g(\frac{2\pi}{15}) + g(\frac{3\pi}{15}) + \cdots + g(\frac{8\pi}{15}) \cdots \frac{2\sqrt{2}}{5} \quad \square \square \square$$

$$10 \square \square \square \square \square \square \quad - \frac{1}{2} x^2 + 1, \quad \cos x \quad \square \quad x \in [-1, 1] \quad \square$$

$$\square \square \square \quad k \ln \sqrt{x^2 + 1} + \cos x - 1, \quad 0 \quad \square \quad x \in [-1, 1] \quad \square \square \square \square \square \quad k \square \square \square \square \square \square$$

$$\square \quad \square \quad \square \quad \square \quad \square \quad \square \quad f(x) = 2 \ln x + x^2 + x \quad \square \quad \square \quad \square \quad \square \quad \square \quad x_1 \quad x_2 \quad \square \quad \square \quad f(x_1) + f(x_2) = 4 \quad \square \quad \square \quad \square \quad \square \quad \square \quad t \in (0, \frac{\pi}{4}) \quad \square \quad \square \quad \square \quad \square$$

$$\cos(\tan t) - \ln(\cos t), \quad \frac{x_1 + x_2}{2} \quad \square$$

$$\square \square \square \square \square \square \square \quad g(x) = \cos \quad x + \frac{1}{2} x^2 - 1 \quad \square \quad x \in [0, 1] \quad \square \square \quad g(x) = -\sin \quad x + x, 0 \quad \square$$

$$\square \quad g(x) \quad \square \square \quad [0, 1] \quad \square \square \square \square \square \square \square \quad g(x) \cdots g(0) = 0 \quad \square$$

$$g(x) \text{ on } x \in [-1, 1] \quad g(x) = \cos \left(x + \frac{1}{2}x^2 - 1 \right) \cdot 0$$

$$- \frac{1}{2}x^2 + 1, \cos x \text{ on } x \in [-1, 1]$$

$$2 \ln \sqrt{x^2 + 1} + 1, 1 - \cos x, \frac{1}{2}x^2$$

$$\ln(x^2 + 1), x^2 \quad \ln(x^2 + 1), x^2 \quad k, 1$$

$$k, 1 \quad \ln \sqrt{x^2 + 1} + \cos x - 1, \ln \sqrt{x^2 + 1} + \cos x - 1$$

$$h(x) = \ln \sqrt{x^2 + 1} + \cos x - 1 \quad h(x) \text{ on } x \in [0, 1] \quad h'(x) = \frac{x}{x^2 + 1} - \sin x$$

$$m(x) = \frac{x}{x^2 + 1} - \sin x \quad m'(x) = \frac{1 - x^2}{(x^2 + 1)^2} - \cos x, -\cos x + 1 - x^2, -\cos x + 1 - \frac{1}{2}x^2, 0$$

$$m(x) \text{ on } x \in [0, 1] \quad h'(x), h(0) = 0$$

$$h(x) \text{ on } x \in [0, 1] \quad h(x), h(0) = 0$$

$$k, 1 \text{ on } x \in [-1, 1]$$

$$k \text{ on } (-\infty, 1]$$

$$3 \quad t \in (0, \frac{\pi}{4}) \quad \tan t \in (0, 1) \quad 2 \quad k = 1 \quad x = \tan t \quad \cos(\tan t) - \ln(\cos t), 1 \quad \frac{x_1 + x_2}{2} \dots 1$$

$$x_1 + x_2 \dots 2$$

$$f(x_1) + f(x_2) = 4 \quad 2 \ln x_1 + x_1^2 + x_1 + 2 \ln x_2 + x_2^2 + x_2 = 4$$

$$2 \ln(x_1 x_2) + (x_1 + x_2)^2 + x_1 + x_2 - 2x_1 x_2 = 4 \quad (x_1 + x_2)2 + x_1 + x_2 = 4 + 2[x_1 x_2 - \ln(x_1 x_2)]$$

$$y = x - \ln x \quad x = 1 \quad (x_1 + x_2)^2 + (x_1 + x_2) \dots 6$$

$$x_1 + x_2 > 0 \quad x_1 + x_2 \dots 2$$

$$\forall t \in (0, \pi/4) \quad \cos(\tan t) - \ln \cos t > \frac{x_1 + x_2}{2}$$

$$0 < x_1, x_2$$

$$f(x) = \frac{2}{x} + 2x + 1 > 0 \quad f(x) \in (0, +\infty)$$

$$f(1) = 2 \quad f(x_1) + f(x_2) = 4 \quad 0 < x_1, 1, x_2 \quad x_1 + x_2 \leq 2 \quad x_2 \leq 2 - x_1 \quad f(x_2) \leq f(2 - x_1)$$

$$f(x_1) + f(x_2) = 4$$

$$4 \leq f(2 - x_1) + f(x_1)$$

$$F(x) = f(x) + f(2-x) \quad x \in (0, 1] \quad F(x) = f(x) + f(2-x) = 4(1-x)\left(\frac{1}{x(2-x)} - 1\right) \geq 0$$

$$F(x) \in (0, 1] \quad F(x), F(1) = 2f(1) = 4$$

$$x = x_1 \quad 4 \leq f(2 - x_1) + f(x_1) \leq f(x_2) + f(2 - x_1) \leq x_2 + 2 - x_1 \leq x_1 + x_2 \leq 2$$

$$\forall t \in (0, \frac{\pi}{4}) \quad \cos(\tan t) - \ln \cos t > \frac{x_1 + x_2}{2}$$

$$f(x) = \sin x \quad g(x) = \ln x \quad h(x) = x^2 - ax - 1$$

$$\forall x \in [0, 1] \quad f(x) \leq g(x+1)$$

$$\forall x \in (0, 1] \quad e^{f(x)} + h(x) - g(x) > 0 \quad a$$

$$f(x) = \sin x - \ln(x+1) \quad (0, x < 1)$$

$$f(x) = \cos x - \frac{1}{x+1}$$

$$f(0) = 0 \quad x \in [0, 1]$$

$$f(x) = \frac{1}{(x+1)^2} - \sin x$$

$$F'(x) \in [0, 1] \implies F'(x) \leq F'(0) = 1 \implies F(x) \leq F(0) + x = 1 + x$$

$$x_0 \in (0, 1) \implies F'(x_0) = 0$$

$$F(x) \in (0, x_0) \implies F(x) \in (x_0, 1)$$

$$F'(1) = -\frac{1}{2} + \cos 1 > -\frac{1}{2} + \cos \frac{\pi}{3} = 0 \implies F'(0) = 0$$

$$F(x) > 0 \implies (0, 1) \implies F(x) \in [0, 1]$$

$$F(x) \leq F(0) = 0 \implies F(x) \leq 0$$

$$f(x) \leq g(x+1)$$

$$2 \implies x \in (0, 1] \implies e^{f(x)} + h(x) - g(x) > 0$$

$$e^{a \ln x} + x^2 - ax - 1 - \ln x > 0$$

$$x=1 \implies e^{a \ln 1} > a$$

$$1 \implies \sin 1 > \ln 2 \implies 2 = e^{\ln 2} < e^{\sin 1} < e^1 < 3$$

$$a \implies e^{a \ln x} + x^2 - ax - 1 - \ln x > 0 \implies a, 2$$

$$e^{a \ln x} + x^2 - ax - 1 - \ln x > e^{a \ln x} + x^2 - 2x - 1 - \ln x$$

$$H(x) = e^{a \ln x} + x^2 - 2x - 1 - \ln x > 0 \implies (0, 1]$$

$$1 \implies \sin x > \ln(x+1) \implies e^{a \ln x} > x+1$$

$$H(x) > x+1 + x^2 - 2x - 1 - \ln x = x^2 - x - \ln x$$

$$G(x) = x^2 - x - \ln x \implies x \in (0, 1]$$

$$G(x) = 2x - 1 - \frac{1}{x} = \frac{(2x+1)(x-1)}{x} \geq 0$$

$G(x) \in (0, 1]$

$G(x) \dots G(1) = 0$

$a \dots 2$

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